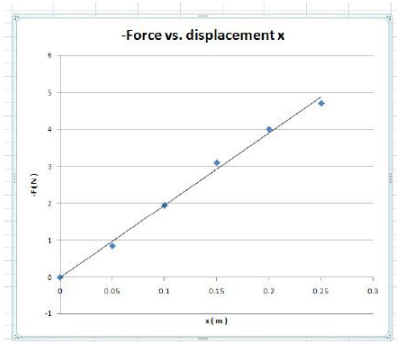
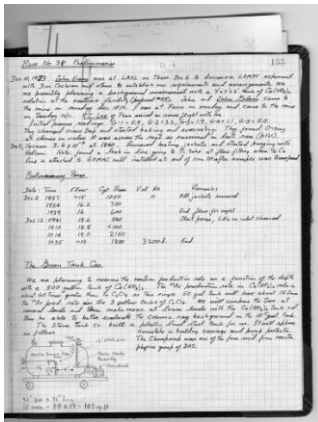


Data Analysis Handbook



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What is data?

Physics is a subject that aims to develop our understanding of the way the Universe works. This is a pretty grand aim, and it is a fantastic achievement that we have managed to learn so much so far. Of course, there is a huge amount still to learn, especially in the field of cosmology.

There are two tools that physicists use to develop our understanding of the way the physical world works.

1. **Experiments.** Physicists probe the physical world by carrying out tests. *What if....?*, *suppose we...?*, *how about...?*, are all questions that lend themselves to practical investigations. In your study of physics you will already have carried out a large number of experiments.

2. **Mathematics.** Of all the science subjects, physics has the strongest relationship with maths. Physicists are comfortable using numbers. We measure things and record our findings in terms of numbers.

In this document we will use the word **data** to describe the numbers that are recorded as we take measurements during experiments.

Data is sometimes referred to as raw data. This gives us a clue as to what it is. Data on its own doesn't tell us much. It is only by analyzing data that we can turn it into information and knowledge.

Scientists collect data – usually by carrying out experiments and taking measurements. They then look for patterns in the data. How do the numbers relate to each other? When one set of numbers increases, what happens to another set? Does it go up – or go down? This is **data analysis**.

Activity:

Consider the following quotation:

“When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind.”

Lord Kelvin 1892.

Do you agree with this? You may like to discuss this in your class.

It is useful to discuss the quotation in the context of other subjects. What do historians or musicians make of it?

Singular or plural?

The word data is a plural word but is commonly used as a singular word.

So, strictly speaking we should refer to one datum – and lots of data. However, it is common to say things like *“the data has been collected”*, instead of *“the data have been collected.”* In this document you will see examples where the word data is treated as a singular word. If you wish, you can mentally correct this!

Reliability – or how do we know our data is good data?

Reliability

What does it mean if someone describes you as being **reliable**?

If you are reliable, you can be trusted. Perhaps you can be relied upon to turn up for classes on time. You do your work consistently well and your friends know that if you say you'll do something, then you will.

On the other hand, someone who is unreliable is inconsistent. They don't always turn up on time and perhaps their work isn't consistent.

Data generated from experiments should be reliable. Suppose we have just carried out an experiment and we have generated some data by taking measurements. How do we know if our data is reliable? Well, we don't necessarily! By undertaking one set of measurements, we don't have any evidence that we can trust our data. How do we know how much trust or confidence we can place in our results? Trusting the results from one set of measurements would be like concluding that a friend is reliable because they have turned up on time once. **To get some idea of how reliable our results are, we need to repeat our measurements.** If we repeat our measurements and get quite different results, then that tells us loudly and clearly that our results are not reliable. On the other hand, if our second (and further) results are the same, or are similar to the first set, then we can start to have confidence that our data is reliable.

It is for this reason that **we should normally repeat our measurements when undertaking practical work in physics.**

Activity

Use the internet to find a conclusion from a piece of physics research that was based on unreliable data.

Hint: try searching for *Famous Experimental Errors*

How many repeat measurements?

How many times should we repeat our measurements? There is no one answer to this. It depends! Sometimes repeating a measurement gives an identical number. In this case, there is no point in repeating measurements more than is required to ensure that the number is not going to change. On the other hand, some measurements will give a different number every time the measurement is taken. In this case, it makes sense to repeat the measurement and find a mean. Five measurements is usual, and an average should never be found from less than three. If the measurements are changing quite a lot, it may be necessary to take more than five measurements.

Are reliable results good results?

Perhaps. Certainly the more times we do an experiment and the more often we obtain the same results, the more confident we can be that our data is reliable.

But what if we had made a mistake in taking our measurements, and what if we kept making the same mistake each time we repeated the measurement? For example, we may have used a thermometer that is faulty or we could have overlooked something that affects our results (see controlling variables). Our data would appear to be reliable. We may have confidence in our results but any conclusion we might make would not be valid (i.e. it would be incorrect). So what to do? Well, the way physics works is that another physicist somewhere else may repeat our experiment. They may get the same results, in which case our confidence in the reliability of our data increases. However, if they get different results, then this would set the alarm bells ringing. In practice, in a learning environment, it is not practicable to ask another student to check our results so, as long as we have tried our best to avoid making mistakes and have used non-faulty equipment, we can be reasonably confident that our data is reliable if we have repeated our measurements a sufficient number of times.

Precision and accuracy – same difference?

Precision and Accuracy

What is the difference between precision and accuracy?

The **accuracy** of a measurement tells you how close the measurement is to the “true” or accepted value.

The **precision** of a measurement tells you something about the **number of significant figures** in the measurement.

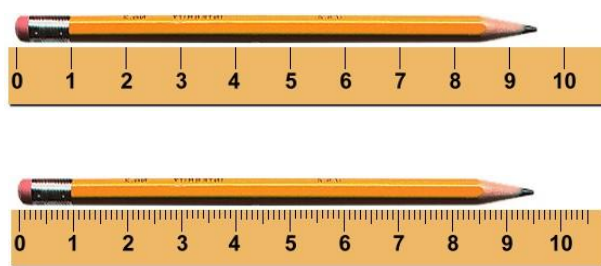
Accuracy

Whatever we are measuring, the accuracy of our measurement depends on the quality of the measuring device we are using.

If you wish to measure your own weight you (probably) want the scales to read your true weight. However, many bathroom scales under read or over read. How can you know how close your scales are to the true value? Well, you can repeat the measurement using a higher quality scale - but even so, how can you be sure that this is more accurate? Ultimately, we can never measure a true value with 100% confidence. Despite this, we should still try to ensure that all our measurements are as accurate as possible. As a student experimenter, you will be restricted in the equipment you will use. Nevertheless, there are things that you can do to improve the accuracy of your measurements. A good start is to ensure the instrument is zeroed and used correctly!



If you only have one watch, you always know what time it is. If you have two watches, you are never quite sure.



Precision

Consider the two rules being used to measure the length of the same pencil. The lower rule allows a more precise measurement to be taken.

The length of the pencil as measured by the upper rule is somewhere between 9 cm and 10 cm. We might guess and say 9.5 cm but the decimal place is just a guess. The smallest unit on this rule is 1 cm.

With the lower rule the length is measured to be 9.5 cm and we might guess at 9.51 cm. The smallest unit on this rule is 1 mm.

The measurement using the lower rule is **more precise because it uses a smaller unit to measure with**.

So how long is the pencil? Well, the answer is we can't really know. We know that the lower rule gives us a more precise measurement but how do we know it is accurate? You might like to check the length of the two rules above. Are they the same length?

Activity – group discussion

Discuss examples of measurements that have high accuracy but low precision.

Discuss examples of measurements that have high precision but low accuracy.

Independent and dependent variables

What is a variable? Simple. It's something that varies. Most of the things we measure in physics are variables.

Physicists have found it useful to describe a large number of properties such as mass, temperature, resistance, density and so on. These properties are variables – that is, they change. Furthermore it seems that if we deliberately change one of the variables, then inevitably another variable gets changed.

For example, you have already discovered that if you change the temperature of a material, the volume also changes (usually). Or if the resistance in a circuit is changed, the current also changes.

Much of physics is about exploring how variables are related to each other. The equations we use to solve problems are statements of the relationships between variables.

If we wish to explore the relationship between two variables, we choose one to vary and see what effect it has on another variable. It is conventional to call the variables we are investigating the dependent and independent variables. How do we remember which is which?

The one that we have **control over**, or is the one that we are increasing or decreasing in steps, is the **independent variable**.

The one that **changes in response** to what we are doing is the **dependent variable**.

The examples below illustrate this.

Controlling variables

When an investigation is being carried out there are many variables that could affect the results. For example, when investigating the relationship between the UV transmitted through suncream of different protection factors, the thickness of the suncream will also affect the measurements. It is important to **control** this variable otherwise we won't know if it is the difference in suncream protection factor, or the thickness of suncream that is having an effect on the UV transmission.

In general, we need to make sure that we control all the variables that may have an effect on our measurements. In your early science studies, you may have called this carrying out a fair test. Controlling variables is one of the hardest things to plan for in carrying out investigations.

Investigation	Independent variable.	Dependent variable.
What is the relationship between the resistance of a semiconductor and its temperature?	We change the temperature in steps.	Therefore the resistance changes.
What is the relationship between the kinetic energy of a moving vehicle and its velocity?	We change the energy of the vehicle in steps.	Therefore the velocity changes
What is the relationship between the angle of incidence and angle of refraction in a glass block?	We change the angle of incidence in steps.	Therefore the angle of refraction changes.

Higher Physics

Data analysis

Significant figures

Calculators are great but.....

Suppose we have done an experiment and our measurements have generated some data.

Voltage = 5.3 V

Current = 1.4 A

Now we wish to calculate the resistance.

Try doing the calculation using a calculator.

The result we get is a number with lots of numbers after the decimal place. How many do we include in our answer?

The answer to this question is that it depends on how precisely the voltage and current have been measured (or stated in the problem).

Consider the voltage 5.3 V. The meter we have used has allowed us to say with confidence that the voltage is 5.3 V, and not just 5 V. However, the meter has not allowed us to state the next decimal place. So, we don't know if the voltage is 5.31 V, or 5.34.

We say that the number 5.3 has two significant figures (sig figs). The measured value of the current also has two sig figs.

Even though a calculator has helped us by carrying out calculations, we still need to think about the answer and ensure we include the correct number of sig figs. Because the calculated value of the resistance depends on the measured values of current and voltage, we would be incorrect to claim any more sig figs in its value.

Our calculated value of resistance is therefore 3.8 Ω .

Significant Figures – the most important rule

In any calculation, the final answer can have no more significant figures than the value used in the calculation which has the LEAST number of significant figures.

Activity – Three of these calculations have an error in the sig figs of the calculated value – Which ones?

a) $\frac{20.4}{3.4} = 6.0$

b) $3.52 \times 9.123 = 32.11$

c) $\frac{0.03}{0.15} = 0.20$

d) $(5.0)^2 \times 0.91 = 22.75$

e) $\frac{0.202}{2020} = 1.00 \times 10^{-4}$

f) $\frac{2.2 \times 10^6}{1.1 \times 10^3} = 2.0 \times 10^3$

How to decide how many significant figures are in a number			
	Rule	Examples	
1	All non zero digits are significant	54 has two sig figs	54.321 has five sig figs
2	Leading zeros are not significant	0.0054 has two sig figs	0.00000054 has two sig figs
3	Zeros appearing anywhere between two non-zero digits are significant	54.03 has four sig figs	0.501 has three sig figs
4	Trailing zeros in a number containing a decimal are significant	2.500 has four sig figs	1.20 has three sig figs
5	Trailing zeros in a number not containing a decimal are ambiguous and should not be written in this way.	3200 could have two or four sig figs. It is much better to use scientific notation. 3.2 x10 ³ has two sig figs 3.200 x10 ³ has four sig figs	

Average or Mean?

Suppose we have carried out an investigation and following good practice we have repeated our measurements to obtain the following data.

9.1 V, 9.3 V, 9.1 V, 9.4 V, 9.4 V, 9.1 V, 9.2 V

There are seven measurements of voltage and we note that they are not the same. How do we know which value to use?

A glance at the numbers suggests that a number somewhere between the largest and smallest is probably a good representation to use. The number we choose is often called the **average**. Perhaps surprisingly, there are a number of ways to calculate this representative number.

In Higher Physics, the type of average that will be used is the **Mean**. (Strictly speaking we use an arithmetic mean.)

The mean is simply found by adding up all the numbers and dividing by the number in the sample.

In mathematical terms this can be written as:

$$mean = \frac{1}{n} \sum_{i=1}^n a_i \quad \text{or simply} \quad mean = \frac{\text{sum of values}}{\text{number of values}}$$

(where the data is $\{a_1, \dots, a_n\}$)

Remember significant figures

The mean of the values shown above is:

$$\frac{9.1+9.3+9.1+9.4+9.4+9.1+9.2}{7} = 9.23$$

If we apply the rules for significant figures, then the final answer for the calculated mean is 9.2.

However, in this case we know that the mean really is a bit more than 9.2 and it is acceptable to add one more significant figure than the rule allows. So, stating the mean to be 9.23 is acceptable. This “breaking of the rule” only applies when the numbers in the sample are quite similar.

How many measurements do we need to take to calculate a mean?

There is no single answer to this question. It depends!

Suppose we set up an experiment, controlling variables carefully, and measure something three times, expecting it to remain unchanged. If the measurements give values of say 10.2, 11.9 and 12.3, calculating a mean from only these three values would not be sensible. More measurements are required.

Suppose on the other hand our repeat measurements give us 6.1, 6.2, 6.2, 6.1, 6.2, it is reasonable to assume that a mean calculated from these five will be meaningful.

Deciding how many measurements to take before calculating a mean is best done by inspecting the data and making a judgment on when sufficient measurements have been taken.

For interest only

Statistics has a powerful tool to allow us to quantify the “spread” of measurements around the mean. This is standard deviation. Note that it is not required for Higher physics. There are many articles published on the internet explaining standard deviation if you are sufficiently interested. Knowing the standard deviation of a set of data allows us to estimate how confident we can be in our calculated mean.

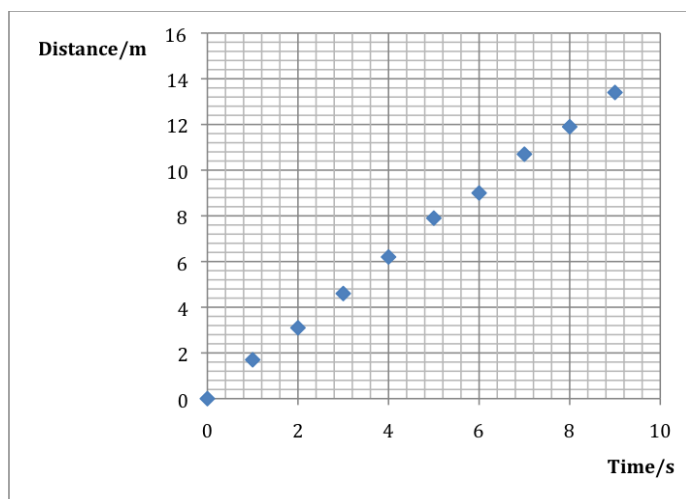
What is a graph?

A graph is simply a visual representation of data. It is easier to understand data when it is presented in graphical form.

There are many different kinds of graphs. The most useful graph used in physics is a **scatterplot** (also called a **scattergraph**). Bar charts, pie charts and histograms are further examples of graphs and although they have their uses, it is not common for them to be used in data analysis in physics.

Drawing and interpreting graphs in physics

An example of a scatterplot, produced using Microsoft Excel is shown below. This graph has been drawn using the data shown in the table.



Hand drawn vs computer generated?

It is important to be able to draw graphs well. There is merit in drawing a graph by hand, particularly in the early stages of developing your skills. By doing this, you will have to think about the scale to be used for each axis, where each point is to be plotted, and you will get a feel for any trend in your graph. However, once you are confident in drawing graphs by hand, it is sensible to use a computer to speed up the process. You still need to consider the scales and labeling of axes, but it is well worthwhile spending time learning how to draw good graphs with a software package such as Microsoft Excel.

Time/s	Distance/m
0	0
1	1.7
2	3.1
3	4.6
4	6.2
5	7.9
6	9.0
7	10.7
8	11.9
9	13.4

IMPORTANT

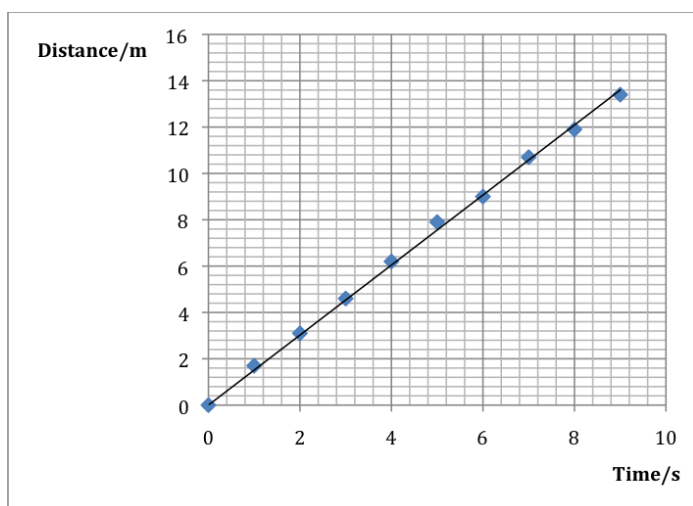
Each column in a table should have a **heading** – which should include a **unit**.

Things to include in a scatterplot

- Labels on each axis. Ultra important! Remember to include the units.
- Values on each axis. You may need to give some thought on the scale to use. Try to use as much as the graph paper as is sensible. If you are using a computer, you have control – don't let the software produce daft scales.
- An origin – usually. More often than not in physics graphs, the origin should be included.
- A title – this is optional. If the title merely states “this is a graph of y against x”, then it can be omitted, as long as the axes are labelled accordingly.
- Data points – obviously. There is no one convention for the symbol to use. A cross, a dot, or a diamond are acceptable. The important thing is that the centre of each point should be obvious.

Trendlines

The scatterplot drawn on the previous page is a visual representation of data that has been collected during an experiment. It appears that as the time increases, the distance gone by whatever was travelling has also increased. Furthermore, the graph shows there is a pattern. The points could be joined by a straight line, although the line may not go through all the points. Remember that all the measurements that have been taken are subject to some **uncertainty**. We can't be sure about the values for each measurement and, taking into account this uncertainty, it is possible to draw a straight line that shows the relationship between the distance travelled and the time taken. The line drawn on the graph is a **line of best fit**. If you use software to draw your graph, this line is called a **trendline**.



Activity

Using the data in the table below draw a scatterplot and include a trendline.

frequency/Hz	Wavelength/m
50	6.8
60	5.7
70	4.9
80	4.3
90	3.8
100	3.4
110	3.1
120	2.8
130	2.6

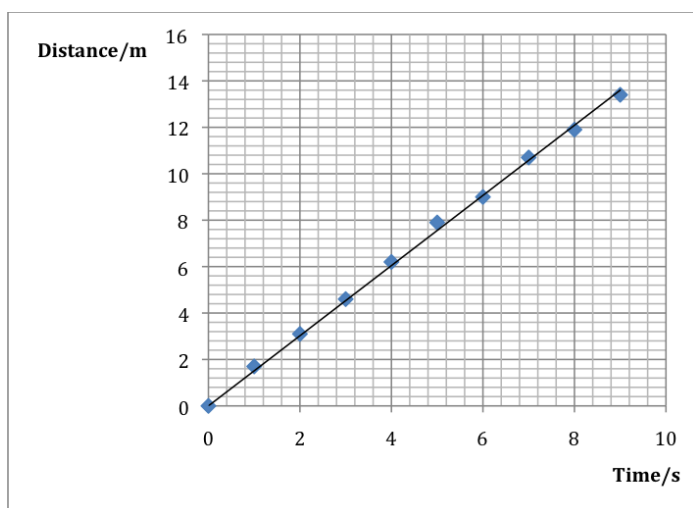
Things to remember when drawing a line of best fit (trendline)

- The line does not need to pass through all the points. In fact, the line may not pass through any of the points. Generally, the line should have an approximately equal number of points on each side.
- The line should only pass through the origin if you really know that it should do. In this case, the moving object really does travel zero distance at time zero, so the origin can be included.
- Many, but not all graphs in physics are straight lines. Trendlines can be curves. If the graph is not an obvious straight line, then don't draw it as a straight line.
- DON'T "join the dots". Drawing graphs in physics is often about establishing relationships between variables. Joining the dots cannot establish a relationship. (Graphs of discontinuous variables are best be represented by a bar chart or similar)

Relationships

As well as giving a visual representation of data, a graph can be used to determine the relationship between variables.

The most straightforward graph to interpret is a straight line graph that goes through the origin. Consider the graph that we met on the previous page.



Here the relationship is that as the time increases, the distance gone increases. We can say that the distance gone is directly proportional to the time.

This can be written as $d \propto t$

where d is distance and t is time

Another way of writing this is $\frac{d}{t} = \text{constant}$

Now $\frac{d}{t}$ can be found from the graph by finding the gradient of the graph. This can be done by calculation, using the maths formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ or if you are using software, by selecting the “display equation” option.

In this case, the gradient is 1.5.

Of course we know that $\frac{d}{t}$ is speed. The speed of the moving object can be determined by finding the gradient of the line of the graph of distance against time. This is a very powerful technique in physics.

Activity

The data in the table below has been collected during an experiment in which the voltage across a resistor and the current through the resistor have been measured.

Draw a graph and use it to determine the resistance of the resistor.

Voltage/V	Current/mA
0.0	0.00
1.5	1.15
3.0	1.94
4.5	2.98
6.0	4.20
7.5	5.05
9.0	5.94

Points to note:

- Although the voltage is the independent variable, in this case it is better to plot voltage on the vertical axis.
- The current is in mA.
- Remember that your final answer needs to take into account significant figures.

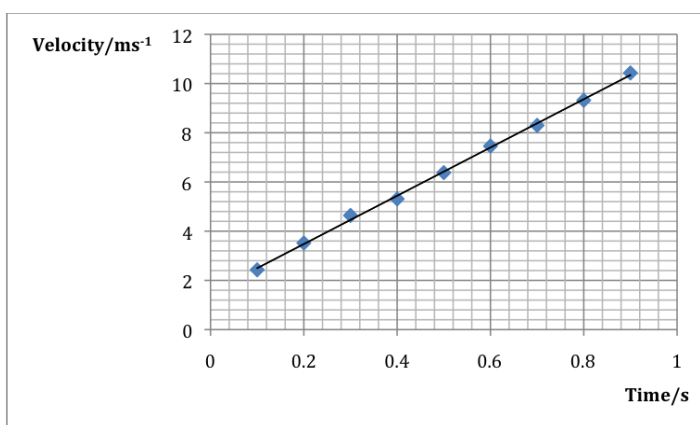
Do we have to draw graphs to find a relationship?

Not necessarily is the answer. In a simple example like the one in the activity above, it would be possible to find the resistance by adding a third column to the table and calculating V/I for each pair of readings. If the values of V/I are constant, or nearly constant, then it is acceptable at Higher level to conclude that the relationship is that V/I is a constant. However, you should not use this method to find the value of the constant (resistance). A graphical method is better because drawing a best fit line allows values of V and I that are measured with less accuracy to be given less weighting.

Interpreting graphs (2)

What about graphs that are not straight line graphs that go through the origin?

Consider the graph below.



This graph is a straight line but the line does not go through the origin.

Recall the equation of a straight line $y = mx + c$ where m is the gradient of the line and c is the point at which the line crosses the y axis.

The gradient of the graph can be found and in this case it is the change in velocity with respect to time. The gradient is the acceleration.

The constant c is the velocity of the moving object at time zero. This can be found by extending the graph back to find the point at which it cuts the vertical axis.

This graph was produced using data for a freely falling object and the object was already moving at zero time.

If you use software to produce the graph, then the equation of the line can easily be displayed on the graph. In this case the equation of the line is $v = 9.8t + 1.5$

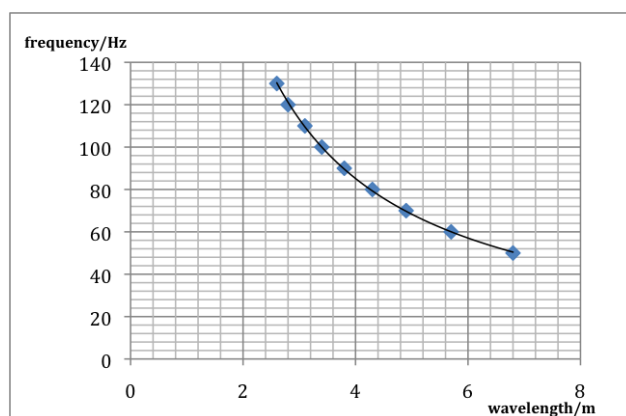
In summary, we have used the graph to find the acceleration of the moving object (9.8 ms^{-2}), and also its initial velocity (1.5 ms^{-1}).

Note that the equation of the line is of the form of the equation of motion $v = u + at$.

Note: a graph that does not go through the origin when it is expected to, is an indication that there are systematic uncertainties in the experimental procedures – see page 17.

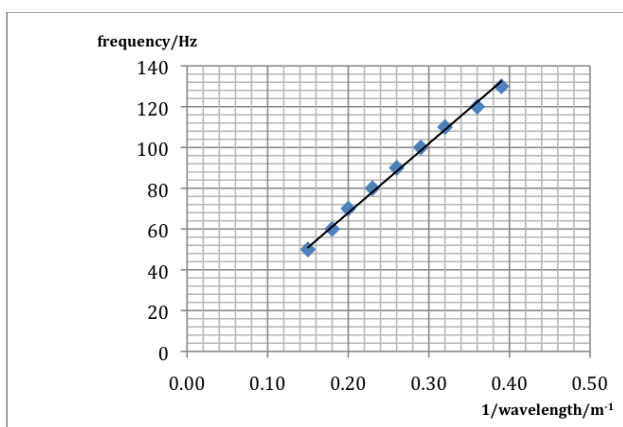
What to do with curves?

Not all data from physics experiments produce straight lines. The data in the activity on page 10 has been plotted on the graph below.



This graph shows an inverse relationship. As one variable increases, the other decreases.

If you suspect your results show an inverse relationship, try plotting another graph of the inverse of the x axis. In this case, try plotting frequency against $1/\text{wavelength}$.



The graph is a straight line that goes through the origin! We can conclude that frequency $\propto 1/\text{wavelength}$

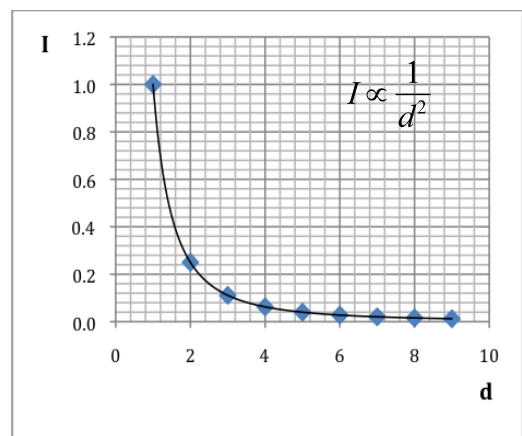
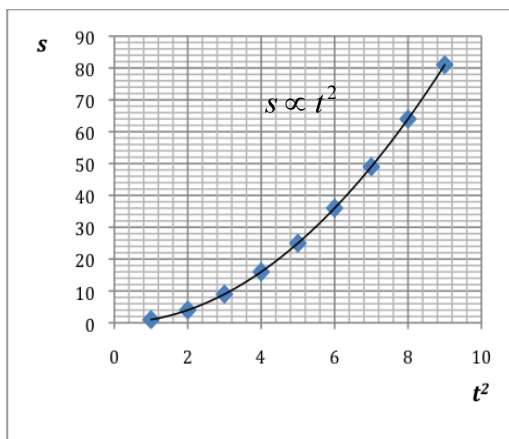
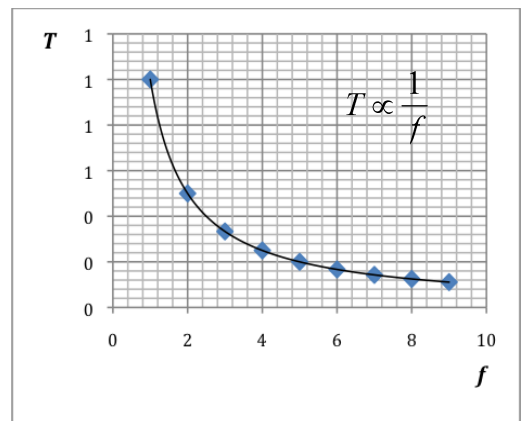
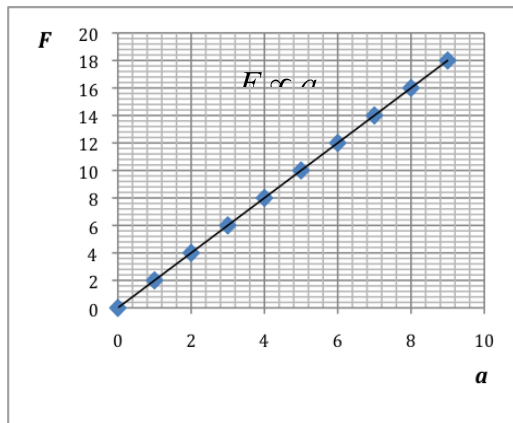
$$\text{or } \frac{f}{\lambda} = \text{constant}$$

In the case, the constant is the speed of the wave.

Interpreting graphs (3)

More curves.

The data from physics experiments can produce a large variety of graphs. This page illustrates a number that you may meet in your work.



Measurement and uncertainty

It is impossible to measure anything with 100% precision. No matter how good our measuring device, and no matter how small the smallest unit on our measuring scale, there will always be an uncertainty in any measurement we make. One of the tasks of a good physicist when they carry out experiments is to make sure they have a good understanding of the uncertainties involved in measurements.

There are a number of ways in which uncertainties can arise in any measurement. These are summarised below and are discussed in detail in the following pages.

Random uncertainties occur when an experiment is repeated and slight variations occur. Random uncertainties can be reduced by taking repeated measurements.

Scale reading uncertainty is a measure of how well an instrument scale can be read. In general, instruments with small unit divisions have a reduced uncertainty.

Systematic uncertainties occur when readings taken are all either too small or all too large. They can arise because of a calibration error or poor experimental design.



Activities

1. Work with a group of students using a rule to measure the width of say a laboratory bench. Each student should measure the width on their own, aiming for as precise measurement as they can. Don't look at each other's measurements until all have completed the task.

How similar or different were the measurements? Can anyone claim their measurement is "correct"?



2. Look at the two tachometers. (Tachometers measure revolutions per minute). Which one would have the largest scale reading uncertainty?



Is the tachometer that allows the most precise measurement to be made necessarily the most accurate? (Refer back to page 5 if you are unsure)

3. Gather a selection of rulers from different manufacturers. Compare their lengths. Are they all identical?

Is there any way of knowing which ruler is best (most accurate)?

A closer look at random uncertainty

Random effects

If a measurement is repeated many times, the result may not be the same each time. Small variations in the experimental conditions or differences in readings taken may result in different values being recorded.

Examples of where random uncertainty occur include:

- Using a stopwatch to measure the period of a pendulum. The reaction time of the person doing the timing may vary slightly each time.
- Measuring a force. The force applied may change slightly each time it is applied.
- Measuring the irradiance of light at distances away from the source. The distance from the light source may be measured slightly differently each time.

Random uncertainty in a mean value

Random uncertainties are equally likely to make measurements higher or lower than the true value. By repeating the measurements, the random uncertainties can be cancelled out by calculating the mean of the readings.

Suppose an experiment to determine the period of a pendulum has been repeated a number of times.

Period in seconds: 0.63, 0.59, 0.57, 0.60, 0.62, 0.59

$$\text{Mean period} = \frac{0.63 + 0.59 + 0.57 + 0.60 + 0.62 + 0.59}{6}$$

$$\text{Mean period} = \frac{3.63}{6} = 0.60 \text{ s}$$

The uncertainty in this value is:

$$\text{uncertainty} = \frac{\text{max} - \text{min}}{n}$$

← IMPORTANT

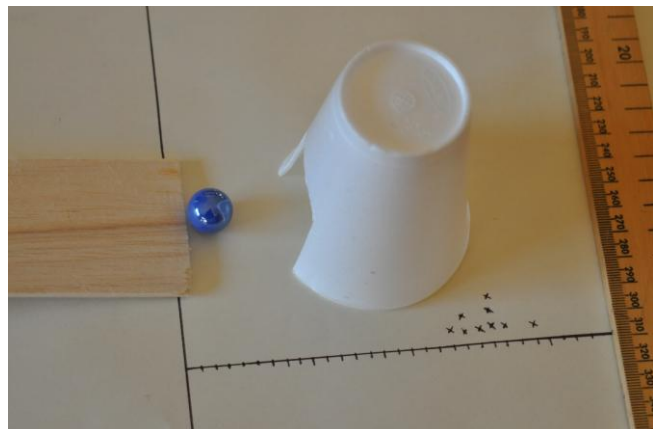
$$\text{uncertainty} = \frac{0.63 - 0.57}{6} = \frac{0.06}{6} = 0.01$$

The final result is: Period = 0.60 ± 0.01 s

Activity

Roll a marble or ball bearing down a slope so that it collides with a polystyrene cup that has a slot cut in it. The cup gets pushed horizontally. Mark the position that the cup gets pushed to by making a cross next to the nearest line. Repeat this until a pattern of crosses is seen.

The spread of distances moved by the cup should be visible. Use the pattern to find the mean distance travelled. The maximum and minimum can be used to find the random uncertainty in this value.



A closer look at scale reading uncertainty

The digital meter shown on the right displays a voltage.
What is the uncertainty in this reading?

As a general rule, the uncertainty is taken as the smallest change that would alter the display. Usually this is the same as saying the uncertainty is 1 in the last digit of the display.

On the voltmeter shown, the display reads 12.88 V. A change of 0.01 V would change the display, so this is the uncertainty in the reading.

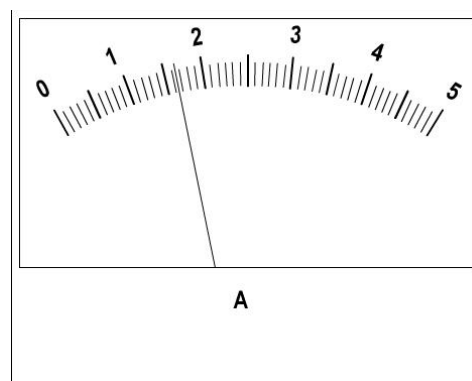


The voltage is 12.88 ± 0.01 V

The analogue ammeter shown on the right displays a current.
What is the uncertainty in this reading?

As a general rule, the uncertainty is taken as half of the smallest division on the scale.

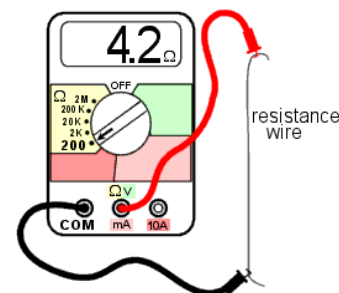
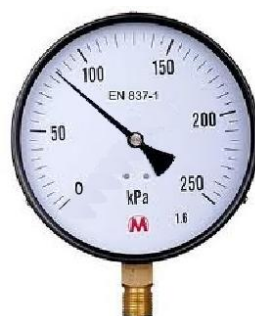
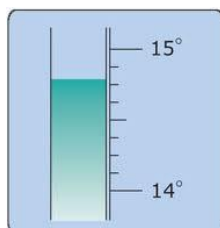
On this ammeter, the needle is between 1.6 and 1.7 so we might reasonably estimate the current to be 1.65 A. The smallest division is 0.1 A. Half of this is 0.05 A so this is the uncertainty in the reading.



The current is 1.65 ± 0.05 A

Activity

Look at each of the scales and state the reading as a value \pm uncertainty.



A closer look at systematic uncertainty

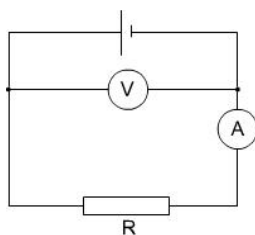
Systematic uncertainty in experimental procedures

Systematic uncertainty occurs when there are faults in the system you use to carry out the experiment. They can be hard to spot and sometimes you may only realize that there are systematic uncertainties in your procedure when you complete your experiment.

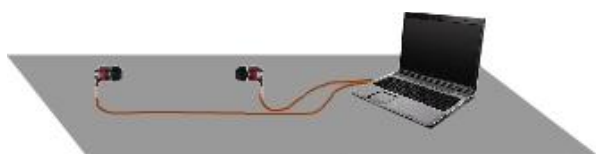
A good way of understanding how systematic uncertainty may occur in experiments is to consider a number of examples.

1. Determining resistance.

The voltmeter in the circuit shown measures the voltage across the supply rather than the voltage across R. The value of R determined by calculating V/I will not be the true resistance of the resistor.



2. Determining the speed of sound in air.



A loudspeaker, microphone and a computer can be used to measure the speed of sound in air. However, in the set up shown, it is likely that the sound will travel through the bench. Sound travels more quickly through a solid, so an incorrect time will be recorded by the computer.

3. Determining the refractive index of a material

The refractive index of a material can be found by measuring angles of incidence and angles of refraction. These angles are measured from a line drawn at right angles to the surface of the material. If this line (the normal) is not drawn correctly, then all the measured angles will have a systematic uncertainty.

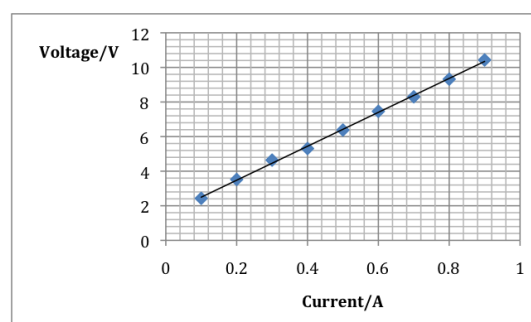
One example of systematic uncertainty in an experiment is using a measuring device that is not calibrated correctly. Examples include:

- A meter rule that is not exactly one meter in length.
- A balance that under or over-reads mass.
- An analogue ammeter that has not had the zero correctly set will consistently read incorrectly.
- A timer that runs slowly.

Although it is true that you will occasionally use measuring devices that introduce systematic uncertainty into your results, it is much more likely that other factors have a greater effect. When evaluating your experimental procedures, try not to fall into the trap of simply stating that the experiment could have been improved by using “better equipment”.

Detecting systematic uncertainty

Systematic uncertainty may show up on a graph of results. If you expect that the graph should go through the origin and it doesn't, then this is a sign that you have something wrong with your experimental setup and you have systematic uncertainty.



Estimating uncertainty in the final result

Absolute and percentage uncertainty

It is important to be able to state an estimate of the uncertainty in any measurement. This can be given as an absolute or percentage uncertainty.

An **absolute uncertainty** is when the uncertainty is expressed as plus or minus (\pm) an estimated amount.

When the result of an experiment is a final result, the uncertainty should be expressed as an absolute uncertainty. Examples include:

$$g = 9.8 \pm 0.2 \text{ m s}^{-2}$$

$$R = 10.4 \pm 0.1 \Omega$$

$$C = 997 \pm 5 \text{ F}$$

A **percentage uncertainty** is when the uncertainty is expressed as a percentage of the measurement.

Examples include:

$$l = 5.6 \text{ m} \pm 4\%$$

$$t = 32 \text{ s} \pm 2\%$$

$$m = 0.75 \text{ kg} \pm 5\%$$

Percentage uncertainties are useful when comparing uncertainties in different measurements.

Combining uncertainties

When a number of measurements are combined to calculate a final result, it is acceptable to assume that the measurement with the largest percentage uncertainty has the greatest effect on the final result and this percentage uncertainty can be used for the calculated result. This is illustrated with the following example.

An experiment to measure the force applied by a hammer uses the relationship $F = \frac{m\Delta v}{t}$ and the following measurements are taken: $m = 0.95 \pm 0.01 \text{ kg}$ $t = 0.0024 \pm 0.0001 \text{ s}$ $\Delta v = 0.43 \pm 0.01 \text{ m s}^{-1}$.

$$\text{The force } F \text{ is } \frac{m\Delta v}{t} = \frac{0.95 \times 0.43}{0.0024} = 170 \text{ N (2 sig figs)}$$

To estimate the uncertainty in this result, first find the percentage uncertainty in each of the measurements.

$$m = 0.95 \pm 0.01 \text{ kg} = 0.95 \text{ kg} \pm 1\% \text{ (approx)}$$

$$t = 0.0024 \pm 0.0001 \text{ s} = 0.0024 \text{ s} \pm 4\% \text{ (approx)}$$

$$\Delta v = 0.43 \pm 0.01 \text{ m s}^{-1} = 0.43 \text{ m s}^{-1} \pm 2\% \text{ (approx)}$$

Here the measurement of the time of impact has the largest percentage uncertainty (4%). Since this is the major contributor to the uncertainty in the force, we can estimate the force to have the same percentage uncertainty.

This gives the result: Force = 170 N \pm 4%

$$\text{Force} = 170 \pm 7 \text{ N}$$

Note. This is simplified method of combining uncertainties. At Advanced Higher, a more sophisticated method is used.

Valid Conclusions

The main aim of undertaking practical investigative work in physics is to enable us to draw conclusions. We do this by making measurements, collecting and analysing data.

Our investigations sometimes allow us to discover or confirm relationships between variables. Sometimes we may find there is no relationship between variables – such a “null result” is just as important as establishing a relationship. On other occasions we may determine the value of a quantity. Examples include finding capacitance, resistance, the acceleration due to gravity, wavelength or refractive index.

Having undertaken our practical work and analysed the data we have collected, it is very important to ensure that the conclusions we draw are valid. When we evaluate our procedures the following will help determine if our conclusions are valid. It may be useful to use them as a checklist when evaluating our experimental procedures.

Have variables that we do not wish to effect the results been controlled?

Has the uncertainty been estimated for all measurements?

Are the results reliable? – i.e. could someone else in another laboratory repeat the experiment and get the same results?

Have the effects of scale reading uncertainty been reduced by using the measuring device with the most appropriate scale?

If a graph has been drawn, are all the points drawn correctly?

Have the effects of random uncertainty been reduced by repeating measurements?

If a trendline is included in a graph, has it been drawn correctly, showing a roughly equal number of points on either side of the line?

Has the percentage uncertainty in all measurements been compared so that the measurement with the greatest uncertainty has been identified?

Have the effects of systematic uncertainty been identified (and reduced if possible)?

If a final result has been calculated, has the absolute uncertainty in the value been estimated?