Determination of Planck's Constant using a Tungsten Filament Lamp

Method and Apparatus

Variations on this experiment have been described by several authors [1], [2], [3], the first of whom attributes the idea to Mr R Giles (who was a physics lecturer at the University of Glasgow in the 1960s). The third source [3] can be found on the internet , giving more of the theory than we do here:

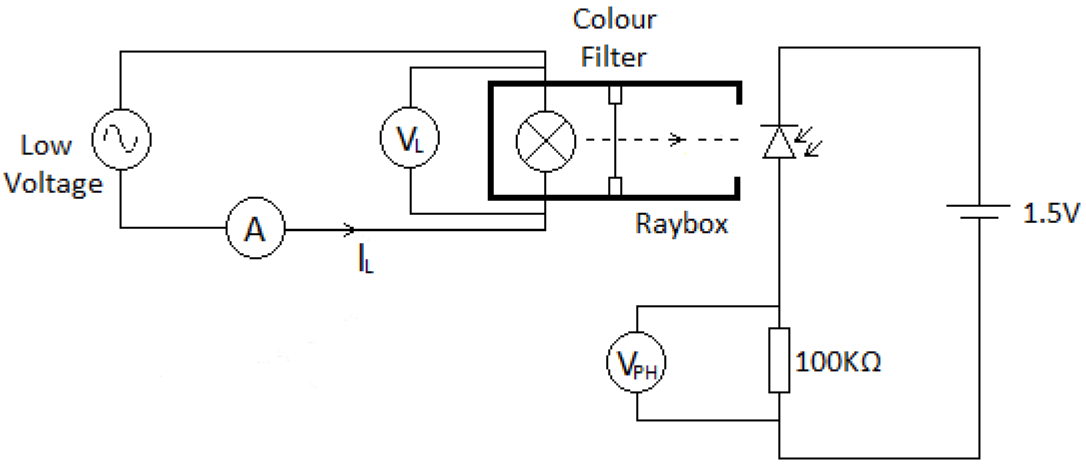
http://www.iitg.ernet.in/scifac/qip/public_html/cd_cell/chapters/p_k_giri_lab_manual/Physics_Lab_manual.pdf

In outline, an energized tungsten-filament lamp (temperature T) is treated as a black-body source of radiation whose emission spectrum peaks in intensity at a frequency determined by the temperature T.

If measurements of intensity are restricted to the visible part of the spectrum, the intensity at any one frequency is a linear function of $1/T$. A value for Planck's constant can be found from the gradient.

Unlike some methods that use mains lamps [1, 3] a 12 V bulb is used, as working at low voltage is much safer. It also means that common lab equipment can be used throughout the experiment.

The equipment is set up as shown, with a raybox powered by a variable low voltage supply. The current (I_L) through the lamp and the voltage (V_L) across it are monitored. The light from the raybox is passed through a colour filter to a photodiode (such as BPW34) in a simple light sensing circuit. The reverse leakage current varies linearly with light intensity. This current is indirectly measured with a voltmeter (V_{PH}) placed across a 100 kΩ resistor in series with the photodiode:



V_L, I_L and V_{PH} are all recorded for a range of raybox voltages from 4V (lamp dim) through to 12V (lamp bright).

Theory

By treating the tungsten filament raybox bulb as a black body radiator the high frequency approximation of Planck's Radiation Law can be applied:

$$U(f) = \frac{8\pi h f^3}{c^3} e^{\frac{-hf}{kT}}$$

U(f) = the energy density with respect to frequency
f = the frequency of light transmitted through the filter
T = the temperature of the filament in Kelvin

(To understand why this approximation holds, at frequencies in the visible radiation spectrum the term (hf/kT) is about 10, which simplifies the Law.)

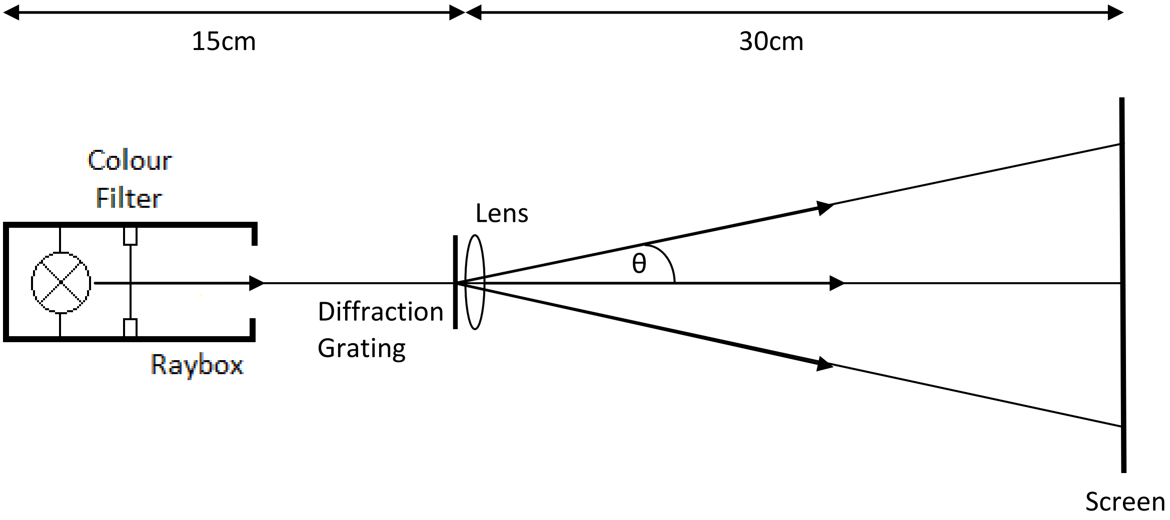
$U(f)$ can be indirectly measured by measuring V_{PH}, since:

$$V_{PH} \propto U(f) \text{ so}$$

$$V_{PH} = A\frac{8\pi h f^3}{c^3} e^{\frac{-hf}{kT}} \qquad \text{where } A \text{ is a constant}$$

taking the natural log: $$\ln V_{PH} = \frac{-hf}{kT} + \ln\frac{A8\pi hf^3}{c^3}$$

Thus the graph of ln V_{PH} against $\frac{1}{T}$ should be a straight line with a gradient of $\frac{-hf}{k}$, where k is Boltzmann's constant. The frequency f can be found using a spectrophotometer, spectrometer, or, if neither is available, diffraction grating. The diffraction grating method involves focusing the light from the filter on a screen using a converging lens (f = 10 cm) such that the lens-to-screen distance is about 30 cm. Place a diffraction grating next to the lens to create a set of interference fringes on the screen. Then, using the geometry of the setup along with $n\lambda = d\sin\theta$ to find λ. f can then be calculated:



Measuring T, the temperature of the filament, is not straightforward. It can be estimated in ^{o}C from the following equation:

$$T = \frac{-\alpha + \sqrt{(\alpha^2 - 4\beta\left(1 - \frac{R}{R_0}\right))}}{2\beta}$$

α and β are constants which for tungsten have the following values: $\alpha = 5.24 \times 10^{-3}\ (^{o}\mathrm{C})^{-1}$, $\beta = 0.7 \times 10^{-6}\ (^{o}\mathrm{C})^{-2}$. R, the resistance of the filament at temperature T (in ^{o}C), can be calculated from the measured values of V_L and I_L. R_0, is the resistance of the filament at 0^{o}C, also has to be found in some way. This is rather tricky to measure for a 12 V bulb as its value is very small. One method is to plot a graph of V against I for the raybox lamp for extremely small voltages and currents and then take the gradient of this graph at the origin as R_0. However, given that the voltages cannot be measured directly across the filament and that the resistances of the connecting wire and contacts are significant compared to the small resistance of the filament at 0^{o}C, it is suspected that the value obtained for R_0 using this method is too large. This is backed up by the fact that using this method to measure R_0 does not yield the best final result for h.

We suggest another way. At the end of the experiment, slowly raise the voltage, monitoring while you do it, until the bulb blows. Apply a rearranged version of the equation above:

$$R_0 = \frac{R}{(1 + \alpha T + \beta T^2)}$$

By noting the current and voltage when the bulb blows, R can be calculated. As this resistance is relatively large the resistances of the connecting wire and contacts are far less significant. T can be assumed to be the melting point of tungsten. This method gives a lower value for R_0 and a better final result for h.

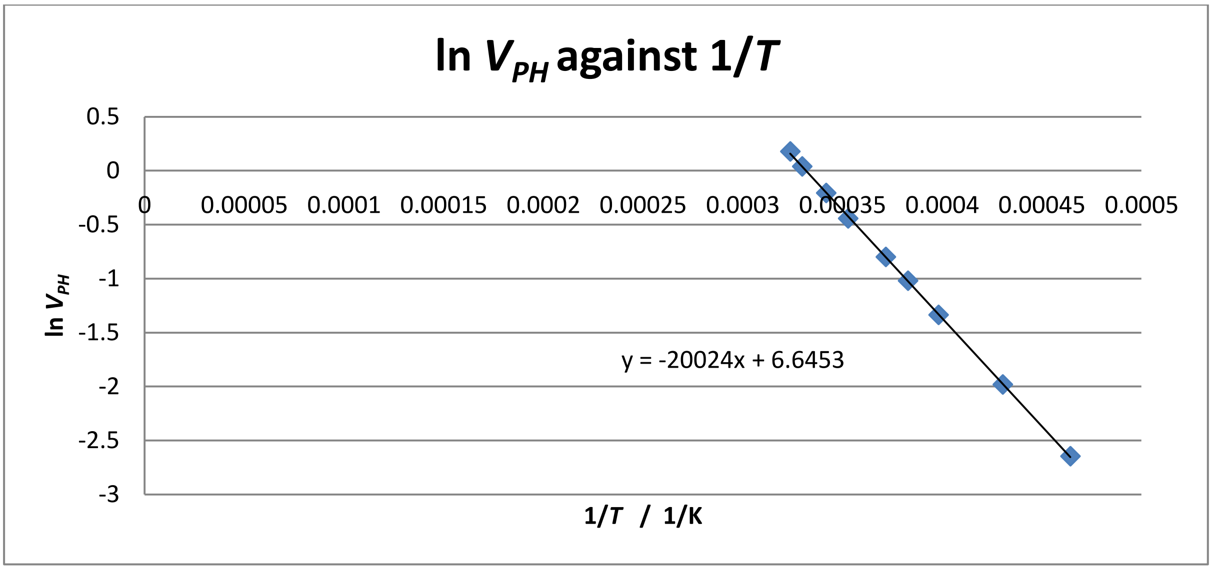
Results

The experiment was performed with a 12 V, 24 W raybox bulb. R_0 for this bulb was determined using the bulb blowing method. This was repeated for a number of these bulbs from the same batch and the results were fairly consistent. The average value of these was 0.29 Ω, and used throughout the calculations. Results for three colour filters (red, green and blue) were obtained. The wavelength of light each of these filters transmits was measured using the diffraction grating method and found to be as follows: Red = 650 nm, Green = 542 nm, Blue = 475 nm.

In the table below are the results from three different runs of the experiment. There is also an example of one of the graphs produced:

	h in x 10^{-34} $\mathrm{m^2kg/s}$		
Filter	Run 1	Run 2	Run 3
Blue	3.96	3.99	4.12
Green	4.72	4.87	4.95
Red	5.87	5.99	5.97
Average	4.85	4.95	5.01

Graph for the red filter, Run 2:



Discussion/Sources of Error

- It was suspected that the colour filters might transmit infrared radiation , which the photodiode in the light sensing circuit would be sensitive to. This was confirmed upon tests with a spectrophotometer. This will obviously lead to an error in the wavelength measurement used in the calculations. Experiments with infrared blocking filters were performed to try and remedy this. However, it seemed to take a combination of three different filters to completely block out all the infrared radiation and produce a significant and consistent improvement in the results. One of these filters came from a webcam, the other two from a specialist optics company. Given that the results obtained without these filters are already within a respectable range of the accepted value of Planck's constant, and that the rest of the equipment used in the experiment is common lab equipment, the additional expense and difficulty of sourcing these filters doesn't seem worth it. However, it should be something any student performing the experiment is aware of.
- As previously mentioned R_0 is a difficult quantity to measure and so this is another important source of error to note, especially since quite small changes in R_0 can lead to large changes in the final calculated value of h.
- It is thought that the bulb-blowing method gives a more accurate value of R_0. One problem with this is that it takes about 22 V to blow a 12 V, 24 W raybox-style bulb. Therefore a low voltage power supply going up to this voltage is needed. Another problem is the non-uniformity of temperature in the filament. If the wire fuses at the hottest part, its average temperature would be less than its melting-point value.
- When it comes to assessing the uncertainty in the results the task is not amenable to algebraic analysis because of the complexity of the relationships. It is suggested that the student sets up a spreadsheet with measured and derived values. The measured values should be tweeked by one or more percentage points and the effect on the derived values found. By working iteratively from value to value, percentage point to percentage point, it should be possible to come to some judgement on the best value for the uncertainty in Planck's constant.

References

1. Meggitt G C 198x The determination of Planck’s constant *School Science Review*
2. Bonnet I, Gabelli J 2010 Probing Planck’s law at home *European Journal of Physics 31 1463-1471*
3. Giri P K 2005 ‘Experiment 16 – Planck’s Constant’ (page 100) Physics Laboratory Manual for Engineering Undergraduates Indian Institute of Technology Guwahati.